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Neurons, Neural Networks and Brain as Objectives of Mathematical Consideration

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Abstract

A new model of neural network was proposed as a type of probable mathematical automaton. The aim of our research was to provide a model of neurons and neural networks that can mimic the properties of actual neurons and nervous systems with sufficient fidelity and comprehensiveness. The model consists of combinations of two elements: neurons and synapses. Neurons receive synaptic inputs and send output action potentials represented by a series of delta function obeying probability density which is a function of synaptic input and internal state of the neuron. Neurons themselves are reset their internal state to the refractory period by the output action potential. Synapses receive inputs from presynaptic neurons and give outputs to post-synaptic neurons as a function of the input and internal state. Internal states of both neurons and synapses change with the pattern and amount of inputs and hence, this model can be used as a realistic learning and self-organizing model.

Keywords: *Automaton; Synapse; Action Potential; Long Term Potentiation; Learning Model*

Introduction

The brain can be regarded as a black box which receives inputs from afferent nerves as a series of action potentials through synaptic connections and sends outputs through the efferent nerves as a series of action potentials. The output response of this black box depends on both the internal state and inputs received. The internal state of the brain includes experience, memory, consciousness, attentiveness and mood.

In the 20th century, with the emergence of new scientific fields including logics, information theory, cybernetics and linguistics, brain function has received a lot of attention, and many notable scientists, including Wiener [1] and Turing [2], conducted studies on theoretical consideration of neurons and brain function. Among these, McCulloch and Pitts in 1943 created simple but effective models of neurons and neural networks [3]. This idea of mathematical automaton was widely accepted and farther researched. In 1959, Shannon and McCarthy edited "Automata Studies" with notable contributors during that period, and the possibilities and limitations of this splendid mathematical device were elucidated [4].

Now, more than 50 years later, computer science has greatly advanced; for instance, artificial intelligence can now defeat world chess masters, and tremendous amount of neuroscientific knowledge has been accumulated.

It is, therefore, necessary to construct a brain model which is sufficiently faithful to the knowledge obtained from neuroscientific researches and to determine the efficiency of such a model to mimic the brain. Here, we provided a brief description of a new type of mathematical automaton that works autonomously and stochastically by receiving action potentials from sensory nerves and gives action potentials through efferent nerves.

Required Properties of the Model

Inputs and outputs of the actual brain or central nervous system (CNS) are received from the afferent nerves and sent through the efferent nerves. All inputs and outputs are conveyed by action potentials, which are generated in all-or-none fashion and carry distinct information based on their timings and intervals (Figure 1). In the model they can be represented by a series of δ-functions because only the timings and intervals of action potentials are necessary to convey the information. Neural network systems, which involve the brain itself or its subsystems, such as hippocampus and cerebellum, consist of neurons that are connected to each other with synapses in a one-way fashion. A neuron receives inputs from synapses and generates action potentials as outputs. These action potentials are conveyed along the neuronal axon and induce the release of neurotransmitters at synaptic junctions which are then connected to dendrites or cell bodies of other neurons. In this model, a neuron and synapse are considered as two different kinds of elements. The neuron has an internal state represented by various parameters that can be changed with time and by the inputs received by the neuron, either slowly or rapidly (Table 1). A synapse also has an internal state; changes in the internal state are important for plasticity of the neural network. Modulatory inputs, such as drugs and nutrients, may directly affect the internal states of elements of the whole brain or its parts.

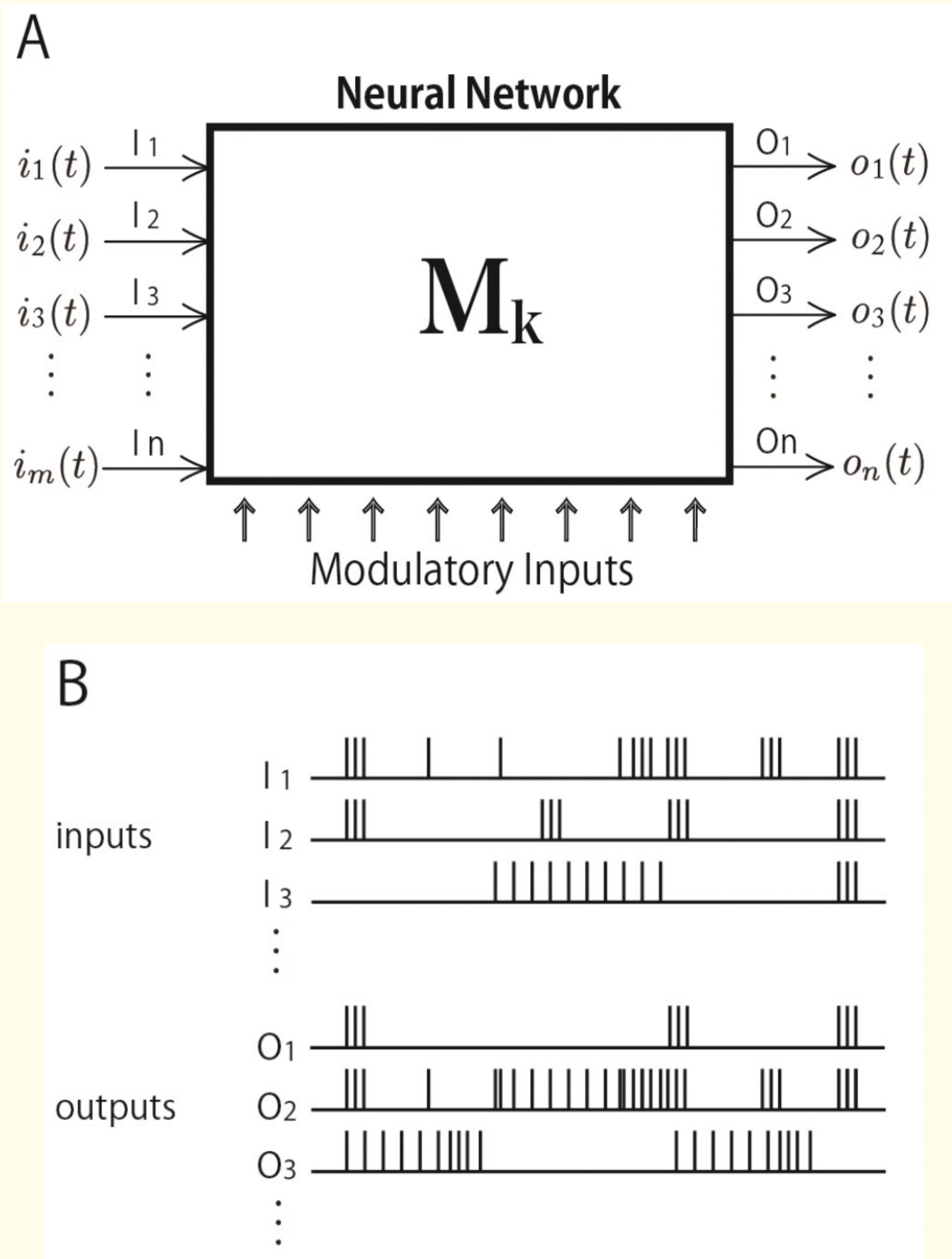

Figure 1: *Inputs and outputs of the neural network A: A neural network (brain or its subsystem) as a black box with connected input and output lines. Input lines represent axons of afferent nerves or neurons outside the network, whereas output lines represent axons from neurons in the network. B: Both the inputs and outputs are action potentials and can be represented by sequences of delta functions. Outputs may correlate with some of the inputs with possible delays (keeping causality). In this example, output* $o_1(t)$ *can be represented as* $i_1(t) \wedge i_2(t)$ *, o2,* $i_1 \vee i_3$ *and* o_3*, ←*i_3*,*

Network or element	Internal state			Input	Output	Modulatory input
	Fixed or almost fixed	Long-term	Short-term and transient			
Brain (CNS)	Character, gender	Learning, experience and ageing	Consciousness and mood	Action potentials	Action potentials	Hormones, ambient transmitters, intrinsic agonist and antagonist, drugs, toxins, nutrients, etc.
Subsystem	Unconditional reflex	Conditional reflex	Activation and suppression	Action potentials	Action potentials	
Neuron	Type of neuron	Differentiation, maturation and ageing	Depolarisation, hyperpolarisation and refractory period	Transmitter release and post-synaptic effect	Action potentials	
Synapse	Transmitter, type of synapse	LTP, LTD and sprouting	Availability of transmitter	Action potentials	Transmitter release and post-synaptic effect	

Table 1

Outline and Mathematical Expression of the Model

Neural Network (Brain or its Subsystem)

Neural networks are clusters of neurons which receive inputs and send outputs (Figure 2). Generally, any combination of neurons with synapses can be regarded as a neural network, even if it does not contain mutually connected parts. The brain or CNS can be considered as a neural network consisting of the whole set of neurons with synapses, which is connected to afferent nerves as input devices and to efferent nerves as output devices.

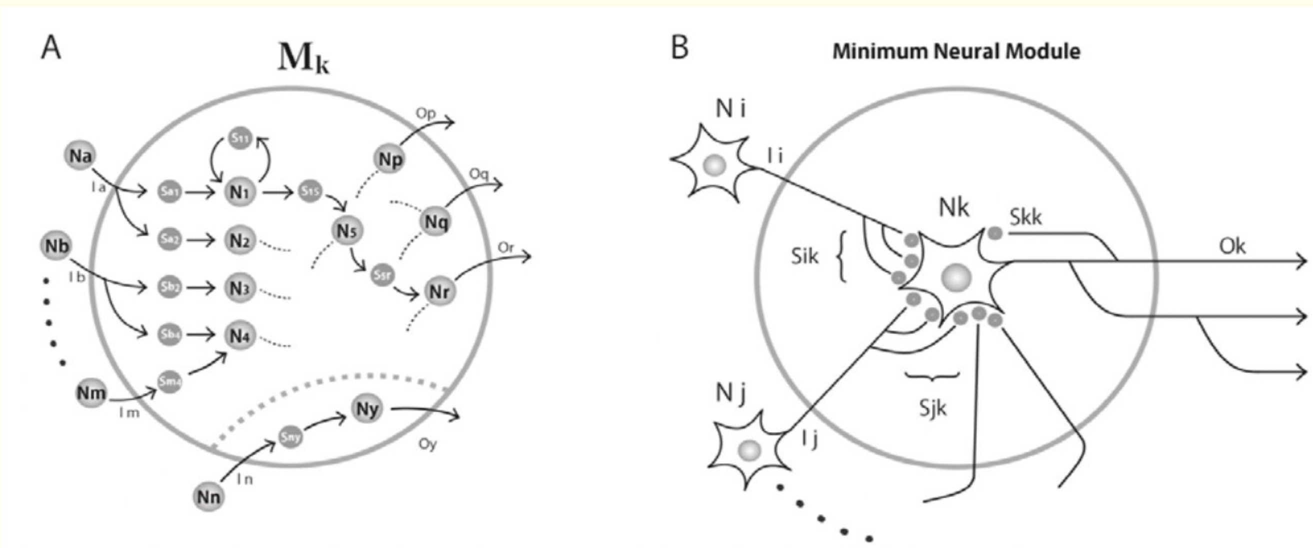

Figure 2: *Elements and connections in the neural network. A: The neural network consisted of neurons and synapses. In this model, neurons and synapses are treated as separate elements of automaton. Synapses which connect two specific neurons are represented by only one element. At the most, one synaptic element (s_{11}) from a neuron to itself is also permissible. Some part of the network might be isolated from the rest as seen in the path from I_n to O_y in this example. B: A neuron can receive inputs from multiple synapses connected from different presynaptic neurons and send to multiple synapses but only one identical output sequence. A neuron with its input-side synapses can be regarded as a minimum neural module.*

The brain, its subsystem and neurons along with synapses which send outputs to the neuron can be regarded as neural networks or modules.

Inputs and outputs of neural network M_k (Figure 1) are expressed as vectors

$$i(t) = (i_1(t), i_2(t), \ldots, i_m(t))$$

$$o(t) = (o_1(t), o_2(t), \ldots, o_n(t))$$

Each input $i_k(t)$ and $o_l(t)$ passes through an afferent axon I_k and efferent axon O_l, respectively.

$i_k(t)$ and $o_l(t)$ are expressed with Dirac's delta function as follows:

$$i_k(t) = \sum_j \delta(t - t_{kj})$$

$$o_1(t) = \sum_j \delta(t - t_{lj}).$$

Internal state s(t) of the neural network is the sum of the internal states of elements and input and output activities in the networks. This also can be expressed as a vector consisting of $s_k(t)$s.

Its behaviour can be formally described by a differential equation as

$$\frac{ds_k(t)}{dt} = G_m(i_1(t), i_2(t) \dots i_m(t), s_k(t), o_1(t), o_2(t) \dots o_n(t))$$

Neuron

Neurons receive inputs from synapses in the form of neurotransmitters which generate excitatory and inhibitory post-synaptic potentials (EPSPs and IPSPs) (Figure 3).

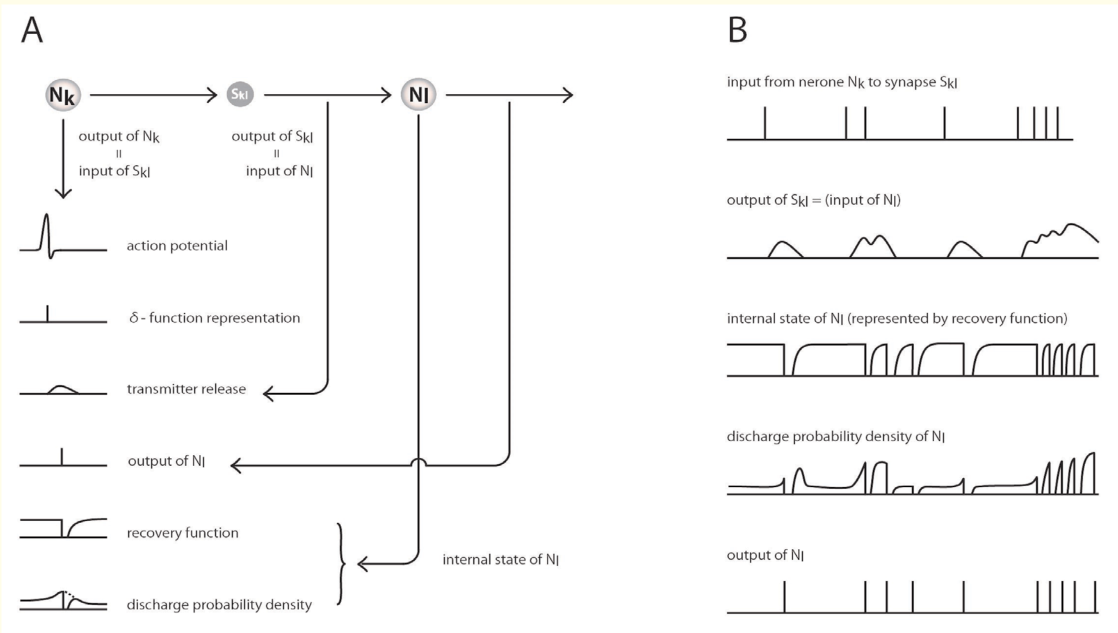

Figure 3: *Example of information transduction in the model: A single action potential output of neuron N_k, represented as a δ-function, causes the release of a neurotransmitter at synapse S_{kl}, which then generates a single action potential output from neuron N_l, as shown in this example. The action potential is stochastically generated by a discharge probability density, which is determined by the internal state of neuron N_l, including the membrane potential shift by EPSP or IPSP and excitability represented by a recovery function. The recovery function itself is reset by the output action potential. B: Output potentials of neuron N_k result in the generation of output potentials of Neuron N_l with slight delays according to the mechanisms shown in A. The timing and number of action potentials, however, can fluctuate.*

The internal state of the neuron includes the membrane potential and proportion of opened ion channels.

The internal state of the neuron N_k is expressed as follows:

$$s_k(t) = (s_{k_1}(t), s_{k_2}(t), \ldots, s_{k_n}(t))$$

It changes according to the following differential equation:

$$\frac{ds_k(t)}{dt} = G_k(o_{a_1k}(t), o_{a_2k}(t), \ldots, o_{a_2k}(t), s_k(t), s_k(t), o_k$$

where $o_{a_1k}(t), o_{a_2k}(t), \ldots, o_{a_nk}(t)$ are outputs of synapses $s_{a_1k}(t), s_{a_2k}(t), \ldots, s_{a_nk}(t)$, respectively.

Outputs of neurons, which are expressed as a series of delta functions, can be determined by the discharge probability density, which is expressed as follows:

$p_k(t) = p_k(s_k(t))$

Output $o_k(t) = \sum_t \delta(t - t_j)$ is determined stochastically using the following equation:

$$P(\exists t_k (t < t_k < t + dt)) = p_k(t)dt,$$

where dt is an infinitely small quantity.

Any neuron N_k can send outputs to multiple synapses which are connected to different neurons, but the output $o_k(t)$ is assumed to be identical, because the effect of delay due to axon length is included in the internal states of the synapses in the model.

Synapse

Synapses act as elemental machines which convert delta functions into transmitter release. All synapses connecting two specific pre- and post-synaptic neurons (N_k and N_l, respectively in figure 3) are represented by one single synapse S_{kl} in this model.

Although not necessarily linear, if the model can be regarded as a linear machine, then it can be expressed with a transfer function $F_{kl}(t)$ as follows:

$$\begin{aligned} o_{kl}(t) &= \int_{-\infty}^{t} F_{kl}(t-\tau) o_k(t) d\tau \\ &= \int_{-\infty}^{t} F_{kl}(t-\tau) \sum_k \delta(\tau - t_k) d\tau \end{aligned}$$

where $o_{kl}(t)$ is the output of synapse S_{kl} which connects the output of neuron N_k to neuron N_l. ok(t) is the output of neuron Nk.

Generally, $o_{kl}(t)$ is expressed as follows:

$$o_{kl}(t) = o_{kl}(s_{kl}(t), o_k(t))$$

where $s_{kl}(t)$ is the internal state of Skl.

The internal state of synapse S_{kl} at time t can be expressed with n parameters as follows:

$$s_{kl}(t) = (s_{kl_1}(t), s_{kl_2}(t), \ldots, s_{kl_n}(t))$$

It changes according to the following differential equation:

$$\frac{d}{dt} s_{kl}(t) = G_{kl}(s_{kl}(t), o_k(t))$$

Discussion

A characteristic property of this neural network model is both its inputs and outputs are in the form of action potentials represented by a sequence of delta-functions. An output action potential is generated according to the discharge probability density, which is determined by the internal state and input received by the efferent neuron. Therefore, the behaviour of this model becomes almost inevitably stochastic. The probabilistic automaton was already studied by von Neumann in the "Automata studies", and was shown to have statistically adequate accuracy within the limitations of information theory [5].

This model has self-similarity. The brain, its subsystems and neurons communicate to the external world via action potentials; however, neurons generate only one identical action potential sequence as the output.

After the proposal of Hebb's Law [6], many neural network models with plasticity of the connectivity, including Perceptron [7,8], Cognitron [9,10], Boltzman machine [11] and Back-propagation [12], have been proposed and used for explaining the mechanisms underlying memory and learning functions of the real brain [13,14].

Action potentials have remarkable characteristics. They are generated in an all-or-none fashion and have identical forms. Hodgkin and Huxley gave a sufficiently accurate mathematical equation for the generation and propagation of action potentials in squid nerves [15]. All action potentials, including those of human brain neurons, can be generated using this equation. The merit of an action potential for information transmission is that it can carry information almost without distortion. Conversely, the generation of action potentials is rather probabilistic and can cause a loss of considerable information. Hence, it is peculiar that the neural machine evolved using this unreliable method, which is one of our main concerns in building this model.

Although we did not mention this in detail in the present model, the brain may be influenced by factors other than afferent axons, such as hormones, drugs, oxygen, glucose and toxins. At the neuronal level, the effects of ambient neurotransmitters or their intrinsic agonist activity through extra-synaptic receptors have recently gathered attention in the field of neuroscience [16].

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Conflict of Interest

The authors declare there is no conflict of interest.

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